April 21, 2013

Dean Henderson, CEO
ABC Oil Company
Salt Lake City, Utah

## RE: Math 1210 Cost Analysis Pipeline Project

Vernal Utah

Dear Mr. Henderson:
On April 21, 2013, Michael Smith of MBS Consulting conducted a cost analysis for the upcoming pipeline project in Vernal, Utah. The purpose of the analysis was to determine the most cost efficient system to run a pipeline from ABC Oil Company's (ABC) newly installed natural gas well to an existing refinery located southeast of the well. Using calculus techniques, multiple scenarios were analyzed to determine ABC's most cost beneficial approach to installing the pipeline.

The scenarios that were analyzed included the following:
Scenario A(1) - Determine the cost of running the pipeline heading east through the mountain and then south to the refinery, staying on Bureau of Land Management (BLM) property the entire way.

Scenario A (2) - Running the pipeline west, south and then east to the refinery, again keeping on BLM property.

Scenario B - Determine the cost of running the pipeline the shortest distance (straight line joining well to refinery across private land).

Scenario C - Determine a cost function that utilizes both private land and BLM land, while not having to head east through the mountain.

Based on the information obtained from this analysis, Scenario $C$ has determined to be the most cost efficient approach to running the pipeline to the refinery. The optimum conditions for this scenario include running the pipeline across private land (a diagonal 6.25 mile section) and then connecting to the refinery on a 16.25 mile section of BLM land (running west to east). The results of the cost analysis for all scenarios are presented in the table below:

| Cost Analysis - Vernal, Utah Pipeline Project |  |  |
| :---: | :---: | :---: |
| Scenario | Cost | Diagram |
| A (1) | \$8,250,000 |  |
| A (2) | \$8,100,000 |  |
| B | \$10,310,000 |  |
| C | \$8,000,000 |  |

Note: All cost data was acquired using mathematical techniques including, but not limited to, the Pythagorean Theorem and Derivative tests to determine the lowest cost distances for Scenario C. For a detailed mathematical interpretation, please refer to the attached calculations sheets. Costs listed in the calculations for pipeline installation, labor, and environmental impact surveys, were provided by ABC.

Based on the information presented in this cost analysis, running the pipeline on both Private and BLM Land presents the most cost effective approach for attaching a pipeline from the newly installed natural gas well to the refinery. The cost analysis equation function along with the graph showing the optimum values for distances verses cost is presented below:

$C(x)=500,000\left(25+x^{\wedge} 2\right)^{\wedge} 1 / 2+300,000(20-x)$

Thank you for allowing us to assist you with this project. If you have questions or comments regarding the information in this report or if we can be of further assistance, please do not hesitate to contact our office at (801) 412-0003.

Sincerely,

## MBS Consulting



Michael B. Smith
Project Analyst

## CALCULATIONS

## Scenerio A (1) - Determine the cost of running the pipeline strictly on BLM ground with two different scenarios:

## Heading east through the mountain and then south to the refinery:

Costs $=\$ 500,000$ mountain drilling costs; $\$ 300,000$ per mile to run pipeline along BLM land ( 25 miles at $\$ 300,000$ per mile to include materials, labor, and fees); $\$ 100,000$ for an Environmental Impact Study; three month project delay at $\$ 50,000$ per month.

Cost Analysis Equation for this scenario:
$C(x)=300,000(x)+750,000(x=$ total miles to run the pipeline $)$

Total Cost $=\$ 8,250,000$


Scenerio A (2) - Heading directly on BLM ground and bypassing the mountain (running west, south, and then east to the refinery:

Costs $=\$ 300,000$ per mile to run pipeline along BLM land (27 miles at $\$ 300,000$ per mile to include materials, labor, and fees).

Cost Analysis Equation for this scenario: $\quad C(x)=300,000(x)(x=$ total miles to run the pipeline $)$

Total Cost $=\$ 8,100,000$


## Scenerio B - Running the pipeline the shortest distance (straight line joining well to refinery across the private ground):

$\underline{\text { Costs }}=\$ 500,000$ per mile to run pipeline directly on private land (\$500,000 per mile to include materials, labor, and fees).

Cost Analysis Equation for this scenario: $\quad C(x)=500,000(x)(x=$ total miles to run the pipeline $)$

To determine the total distance to run the pipeline in this scenario, the Pythagorean Theorem is used based on the sides formed by the BLM land distances and the unknown distance (hypotenuse) through the private land.
$\mathrm{A}^{2}+\mathrm{B}^{2}=\mathrm{X}^{2}$; where $\mathrm{A}=20$ miles; $\mathrm{B}=5$ miles; and $\mathrm{X}=\left(\mathrm{A}^{2}+\mathrm{B}^{2}\right)^{1 / 2}$. Based on the calculation, the total distance of X is equal to 20.62 feet

Total Cost $=\$ 10,310,000$.


## Scenerio C - Finding the lowest optimal cost by running the pipeline on both the Private and BLM land:

Costs $=\$ 500,000$ per mile to run pipeline directly on private land and $\$ 300,000$ per mile to run the pipeline to the refinery on BLM land (costs include materials, labor, and fees). Based on the use of the Pythagorean Theorem, the cost function will be set up as follows (note: refer to the diagram following the calculation basis for triangle information):
$C(x)=500,000(L)+300,000(20-x)$
$\mathrm{L}=\left(25+\mathrm{x}^{2}\right)^{1 / 2}$
$C(x)=\$ 500,000\left(25+x^{2}\right)^{1 / 2}+\$ 300,000(20-x)$

The use of the first derivative test is now used to find the critical point of the function that will determine the minimal distances that the pipeline will be ran through both the Private and BLM Land.
$C^{\prime}(x)=\$ 250,000\left(25+x^{2}\right)^{-1 / 2}(2 x)-300,000 \quad C^{\prime}(x)=\$ 500,000(x)\left(25+x^{2}\right)^{-1 / 2}-300,000$
The first derivative test is utilized by setting this derivate function equal to zero and determining the minimum value for x .

| $\$ 500,000 x\left(25+x^{2}\right)^{-1 / 2}$ | $\$ 300,000=0$ | $\$ 500,000 x /\left(25+x^{2}\right)^{1 / 2}=\$ 300,000$ | $\$ 500,000(x) / \$ 300,000=\left(25+x^{2}\right)^{1 / 2}$ |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
| $\$ 5(x) / \$ 3=\left(25+x^{2}\right)^{1 / 2}$ | $\$ 25 x^{2} / 9=\left(25+x^{2}\right)$ | $25 x^{2}=9\left(25+x^{2}\right)$ | $25 x^{2}-9 x^{2}=225$ | $16 x^{2}=225$ |
| $X^{2}=14.0625$ | $\underline{X}=3.75$ |  |  |  |

This value for $x$ is now put into the Pythagorean Theorem to determine the value $L$ that optimizes the cost for the distance along the Private Land.
$5^{2}+3.75^{2}=L^{2} \quad \underline{L}=6.25$
The following values of $\mathbf{X}=\mathbf{3 . 7 5}$ and $\mathbf{L}=\mathbf{6 . 2 5}$ can now be inserted into the developed cost function to determine the optimal cost for the project:
$C(3.75)=500,000(6.25)+300,000(20-3.75)=\$ 8,000,000$.

As the data indicates, the most cost efficient way to run the pipeline would be Scenario C.


$C(x)=500,000\left(25+x^{\wedge} 2\right)^{\wedge} 1 / 2+300,000(20-x)$

## Reflective Writing

When I first began Math 1210, I did not realize the practical uses that it brings to real world situations. The last time I had taken calculus was in High School. I placed out of calculus when I received my original degree from Utah State University in Science. I can honestly say that I did not remember the techniques that I would be using to succeed during the upcoming semester. Working on the pipeline project was an eye opener for me in regards to the usefulness of calculus in conducting cost analysis. With simple derivative tests and cost function generation, I was decisively able to determine the most cost efficient way to run the pipeline. I feel the techniques that I have learned during this assignment will help me with future home renovation projects. I am comfortable with using calculus and material costs to estimate the most cost efficient way to conduct future projects.

The U.S. Interior Secretary recently approved drilling of natural gas wells near Vernal, Utah. Your company has begun drilling and established a high-producing well on BLM ground. They now need to build a pipeline to get the natural gas to their refinery. While running the line directly to the refinery will be the least amount of pipe and shortest distance, it would require running the line across private ground and paying a right-of-way fee. There is a mountain directly east of the well that must be drilled through in order to run the pipeline due east. Your company can build the pipeline around the private ground by going 1 mile directly west and then 5 miles south and finally 21 miles east to the refinery (see figure below). Cost for materials, labor and fees to run the pipeline across BLM ground is $\$ 300,000$ per mile. For any pipeline run across private ground, your company incurs an additional $\$ 200,000$ per mile cost for right-ofway fees. Cost of drilling through the existing mountain would be \$500,000 on top of the normal costs of the material, labor and fees for the pipeline itself. Also the BLM will require an environmental impact study before allowing you to drill through the mountain. Cost for the study is estimated to be $\$ 100,000$ and will delay the project by 3 months costing the company another $\$ 50,000$ per month. Your company has asked you to do the following:
a) Determine the cost of running the pipeline strictly on BLM ground with two different scenarios:

1. heading east through the mountain and then south to the refinery
2. running west, south and then east to the refinery.
b) Determine the cost of running the pipeline the shortest distance (straight line joining well to refinery across the private ground).
c) Determine the cost function for this pipeline for the configuration involving running from the well across the private ground at some angle and intersecting the BLM ground to the south and then running east to the refinery. Use this function to determine the optimal place to run the pipeline to minimize cost. Clearly show all work including sketching the placement of the optimal pipeline. Make it very clear how you use your knowledge of calculus to determine the optimal placement of the pipeline. Draw a graph of this cost function and label the point of minimum cost.


Write up a report of your findings to submit to your company's CEO. This report should include solutions to parts a-c showing all math steps and clearly labeled. Summarize the costs to be incurred by each scenario. Include any appropriate figures to make each scenario clear.

Write a paragraph reflecting on the things you have learned in your calculus class and how they may apply to the real world. Do you see calculus as a useful tool? What kinds of things have you learned that can be useful in your areas of interest? Please be specific and give some examples to back up your statements.

